HYDRODYNAMICS OF TUBES OF VARYING CROSS SECTION

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A method is suggested of reducing the hydrodynamic resistance by replacing circular cylindrical tubes (CT) by socalled asymmetrical wavy tubes of varying cross section with long exist cone and short nozzle portions. Mathematical simulationof laminar motion of an incompressible fluid (the Navier–Stokes equations) has shown that a change in geometric parameters can change the resistance substantially, making it larger or smaller than the resistance of an ordinary cylindrical tube. The velocity and pressure fields, friction resistances, pressure profiles, and total resistances of such tubes are given. An experiment has been carried out. A numerical experiment provides a resistance reduced by up to 80%, while the physical experiment provides a reduction by 50%.

A substantial fraction of the total energy generated by mankind is spent on overcoming the hydrodynamic resistance during motion of liquids and gases along tubes. Such losses occur in main pipelines (petroleum, gas, gasoline, ammonia, and so on), in thermal and gas pipelines, aqueduct networks of cities, rural areas and enterprises, and, finally, in element tubes of all machines and structures.

Attempts of loss reduction have so far not been very successful. The use of small amounts of polymers and surface-active materials (SAM), which were investigated for many years at the Donetsk State University [1], gave fairly good results under laboratory conditions (loss reduction by up to 60%) and for a commercial pipeline of diameter 300 mm and length 9 km (reduction by 50%). Due to lack of required SAM the method has not been extended.

1. General Considerations. Qualitative Estimate of Effectiveness. In what follows we consider the method suggested by us [4], valid for both liquids and gases. This method consists of replacing cylindrical circular tubes with a constant transverse cross section, and consequently with constant discharge and maximum velocities over length, by a tube with a periodically varying transverse cross section, and consequently a discharge with maximum velocity. Tubes of Varying Transverse Cross Section (TVC) can be called wavy or asymmetrically wavy.

Theoretically, such a periodic flow can create either a corresponding variation of the tube radius, or an insertion core in a constant radius tube, with a transverse cross section varying with length. The second tube structure is very complicated to prepare, and, obviously, has a very substantial surface moisture, resulting in high friction losses in comparison with an ordinary tube. In this case, when coaxial tubes are used (in drilling instruments, in "tube within a tube" heat exchangers, and so on) it is advisable to enhance their effectiveness by replacing the central tube by a TVC.

In our problem we consider tubes of varying transverse cross section (Fig. 1) with a periodically varying radius. These tubes consist of a set of an infinite number of Venturi tubes. It is seen from Fig. 1 that the basic geometric parameters of a TVC are: the wave length l_0, the exit cone and nozzle portions of length l_e and l_n, the amplitude a, and the radius r_0 of the Cylindrical Tube (CT). In dimensionless form they are:

$$\bar{l}_0=\frac{l_0}{2r_0};\quad \bar{l}_e=\frac{l_e}{l_0};\quad \bar{l}_n=\frac{l_n}{l_0};\quad \bar{a}=\frac{a}{r_0}.$$

The radius r_t of a tube of varying cross section can be represented in the form:

$$r_t=r_0+a\varphi(x), \tag{1}$$

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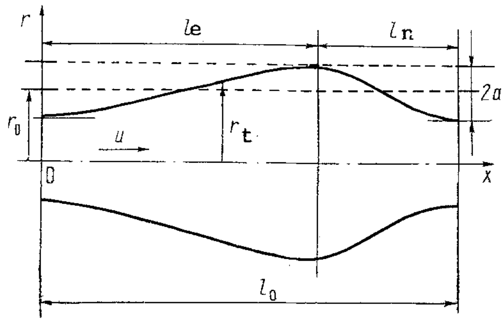

Fig. 1. Geometric characteristics of TVC.

where r_0 is the radius of a cylindrical tube, equivalent to a tube of varying cross section with the same surface area and the same length l_0, and $\varphi(x)$ is a sign-varying periodic function with period l_0. For $x = nl_0$, $\varphi = 1$ and for $x = nl_e$, $\varphi = -1$, while $\int_0^{l_0} \varphi(x)dx = 0$. In principle $\varphi(x)$ must be a continuous function of the coordinate x, having a finite derivative at all points.

In dimensionless form Eq. (1) is written in the form

$$\bar{r}_t = 1 + \bar{a}\varphi(x). \tag{2}$$

In our first theoretical and experimental studies [2] the generating TVC was selected to be a harmonic function

$$\varphi(x) = \cos\left(\frac{2\pi x}{l}\right). \tag{3}$$

Consequently, for x varying from zero to $x = l_e$ we have exit cone flow, and in the x region from l_e to $x = l_0$ – nozzle flow.

The selection of this $\varphi(x)$ for the early studies is explained by its relative simplicity and by the reasoning that if satisfactory effects are obtained for this hydrodynamically incomplete enclosure, then the possibility opens up of substantial enhancement of the effect by shape improvement of the function $\varphi(x)$

It is well known that in the exit cones ($U' < 0$) the friction is always smaller than in a CT, and theoretically one can obtain an exit cone with vanishing friction. In nozzles ($U' > 0$) the friction is larger than in a CT. Consequently, varying the length of the exit cone and nozzle parts of the TVC, one can control friction losses, making them larger or smaller than in an ordinary tube.

We note that when the velocity profiles, and consequently the discharge and maximum velocity $U_m(x)$, are not constant along the tube length, energy losses are generated in the tube besides the loss due to friction, consumed by the deformation in the velocity profiles. These are the so-called pressure or shape losses, or their corresponding resistances.

These losses are usually large for suddenly expanded and constricted flows, i.e., in the presence of interrupted flows. In our problem we treat only uninterrupted flows, and the fraction of shape loss is small.

We carry out initially a qualitative, quite coarse, estimate of the possibilities of loss control during motion of liquids and gases in a TVC. For this purpose we further consider friction and shape losses.

The total resistance, equal to the friction resistance in a CT, is:

$$X_0 = \pi r_0^2 \Delta P_0 = 2\pi r_0 l_0 \tau_0. \tag{4}$$

The friction resistance of a TVC is

$$X_f = 2\pi \int_0^{l_e} \tau_e r dx + 2\pi \int_{l_e}^{l_0} \tau_n r dx, \tag{5}$$

where the subscripts 0, e, and n denote the corresponding values for a CT, and the exit cone and nozzle portions of a TVC.

TABLE 1. Ratio of Friction Forces in TVC and CT

$\bar{l}_e$	0,2			0,4			0,6			0,8			0,9		
K_2	1,2	1,4	1,6	1,2	1,4	1,6	1,2	1,4	1,6	1,2	1,4	1,6	1,4	1,6	2,0
$\bar{X}_f$	0,96	1,12	1,28	0,72	0,84	0,96	0,48	0,56	0,64	0,24	0,28	0,32	0,14	0,16	0,20

If the mean values of friction stresses in the exit cone and nozzle are represented in terms of the corresponding values in a cylindrical tube $\tau_\omega = \tau_0$ in the form $\tau_e = K_1\tau_0$ and $\tau_n = K_2\tau_0$, we then obtain approximately the ratio of the total friction resistance of a TVC to the total resistance in a CT, equal to

$$\bar{X}_f = \frac{X_f}{X_0} = K_1 \frac{l_e}{l_0} + K_2\left(1 - \frac{l_e}{l_0}\right) = (K_1 - K_2)\bar{l}_e + K_2. \tag{6}$$

The values of these coefficients can be estimated from results of numerical calculations.

In the total absence of friction in the exit cone part of the tube (an ideal exit cone) $K_1 = 0$, and

$$\bar{X}_f = K_2(1 - \bar{l}_e) = K_2\bar{l}_n.$$

The ratio of friction resistance in a TVC to the total resistance (due to the same friction) of a CT for various K_2 and $\bar{l}_e$ values is presented in Table 1. It follows from the table that the length variation of the exit cone part of a TVC can be controlled substantially by the friction resistance. Keeping in mind the Reynolds analogy, we assume that in these tubes one can control both heat- and mass-transfer.

We further consider the estimate of pressure or shape losses. Since these losses are substantial for sudden expansions and constrictions, the value of the loss coefficient refers to the kinetic energy either at the inlet or at the outlet. We need to determine the fraction of these losses in total losses of a TVC of length l_0. The shape resistance coefficient is then

$$\lambda_e = \zeta(n)\frac{r_0}{l_0}, \tag{7}$$

where $\zeta(n)$ is the loss coefficient, depending on the extent of compression for a TVC, equal to

$$n = \frac{(r_0 + a)^2}{(r_0 - a)^2} = \frac{(1 + \bar{a})^2}{(1 - \bar{a})^2}.$$

Since $\bar{a}$ is usually less than 0.1, n is small and so is $\zeta(n)$. In a cylindrical tube $n = 1$, $\zeta(n) = 0$, and $r_0/l_0 = 0$.

From the analysis presented it follows that in the limiting case in which the whole length of the selected portion of the tube consists only of the exit cone $\bar{l}_e = 1$, in which friction is absent and instead of the nozzle portion sudden constriction takes place, a staggered tube is obtained, whose resistance is determined only by the shape resistance, i.e., $x_f = x_e$.

Since the resistance of a cylindrical tube is proportional to $\tau_0 l_0$, while the resistance of a staggered tube can be calculated by the Bord equation, one can always select a length l_0, for which the resistance of the staggered tube is smaller than the cylindrical resistance.

Consequently, for motion of liquids and gases in tubes of circular transverse cross section there exist two limiting cases: 1) cylindrical tubes (of constant transverse cross section), whose hydrodynamic resistance is due to friction only; 2) tubes of varying transverse cross section, consisting of an ideal exit cone (friction is absent) and a sudden constriction, staggered tubes whose total resistance is determined only by the shape resistance.

Naturally, there exists an infinite manifold of tubes of varying cross section, whose resistance is determined by the friction resistance and the shape (pressure) resistance. Varying only the lengths of the exit cone and nozzle portions for the corresponding choice of r(x), one can decrease or increase substantially the hydrodynamic resistance (and consequently, the heat transfer) in comparison with the hydrodynamic resistance of a CT, having the same surface area as the TVC. In the limiting case one can obtain a staggered TVC, in which the energy losses are smaller than in the equivalent circular cylindrical tube.

Below we provide a numerical solution and a physical experiment, confirming the qualitative analysis provided.

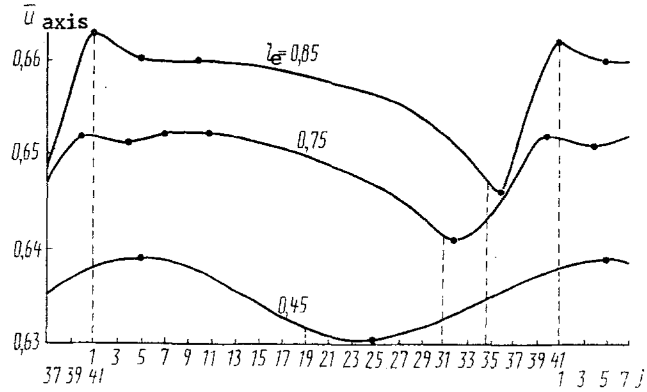

Fig. 2. Velocity variation along the axis for tubes with varying $\bar{l}_e$ at $\bar{a}$ = 0.02; $\bar{l}_0$ = 4.13; Re′ = 1100.

2. Mathematical Model. Velocity and Pressure Fields. As a test model we considered [3, 4] laminar motion of an incompressible fluid, determined by the Navier–Stokes and continuity equations in cylindrical coordinates in the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial x^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right), \tag{8}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v}{\partial r^2}+\frac{\partial^2 v}{\partial x^2}+\frac{1}{r}\frac{\partial v}{\partial r}\right), \tag{9}$$

$$\frac{1}{r}\frac{\partial (rv)}{\partial r}+\frac{\partial u}{\partial x}=0 \tag{10}$$

with the usual boundary conditions: for $r = r_t$ at the wall $u = v = 0$; for $r = 0$ on the axis $v = 0$, $\partial u/\partial r = 0$.

Unlike stabilized motion in a circular tube, for which no transverse velocity exists, while the longitudinal component $u(r)$ depends on the radius only and the pressure on x only, i.e., $\partial p/\partial r = 0$, in a tube with a periodically varying transverse cross section $u(r, x) = u(r, x + l_0)$, $v(r, x) = v(r, x + l_0)$, while for the pressure $p(r, x)$ and the derivative $\partial p/\partial x(r, x) = \partial p/\partial x(r, x + l_0)$.

As usual, to obtain the velocity fields u and v numerically one writes Eqs. (8)-(10) in vorticity ω and stream function ψ variables, to find the pressure field one applies the divergence operation to Eqs. (8) and (9), and, combining them, Poisson equations are obtained. The three equations with correspondingly varying boundary conditions are adequate for numerical integration of any contour of the TVC.

To provide universal boundary conditions by replacing the variable r by $\bar{\eta} = r/r_t$ all possible TVC contours were transformed to a rectangular region. The transformed equations with the corresponding boundary conditions for the rectangular region were carried out by numerical integration with a finite-difference method [3, 5].

As a result of the calculation we obtained: the stream function ψ, the vorticity ω, the velocity fields u, v and the pressure field p in a wide range of geometric characteristics ($\bar{l}_0$ = 2; 4.13; 10; 20; $\bar{l}_e$ = 0.3-0.9; $\bar{a}$ = 0.01-0.05) for sinusoidal formations and Reynolds numbers Re′ = 500-1700.

Not considering the stream function and vorticity fields obtained in the present paper, we provide only several calculation results of the velocity and pressure fields.

Figure 2 shows the variation with length in the dimensionless velocity along the axis for the geometric exit cone portions $\bar{l}_e$ = 0.45, 0.75, and 0.85. It is seen from analyzing the curves that, first, the extreme velocity values usually do not coincide with the geometric extrema. Thus, the minimum velocity points are displaced from the flow relative to the points of maximum transverse cross section for all $\bar{l}_e$. This displacement decreases in this case with increasing $\bar{l}_e$. Second, despite that, the total length of hydrodynamic exit cone portions is somewhat larger than the corresponding geometric lengths. Thus, for $\bar{l}_e$ = 0.45 it is approximately 0.5, and for $\bar{l}_e$ = 0.85 – around 0.9. Assuming that the larger the exit cone length the smaller the friction, it then follows that for $\bar{l}_e$ = 0.85 we have the smallest friction resistance. This indeed occurred in the subsequent

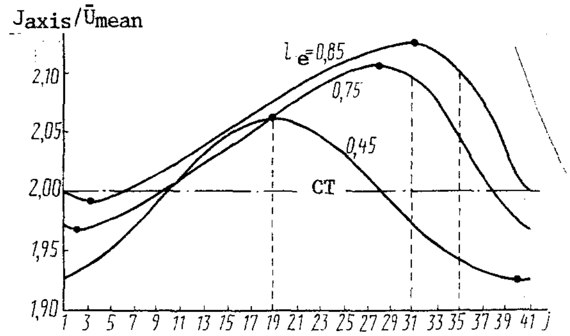

Fig. 3. Distribution of the ratio u_{axis}/u_{mean} over the length of the exit cone–nozzle section with constant $\bar{l}_0 = 4.13$; $\bar{a} = 0.02$; Re' = 1100 for various $\bar{l}_e$: the dashed-dotted line is the ratio $u_{axis}/u_{mean} = 2$ for a CT.

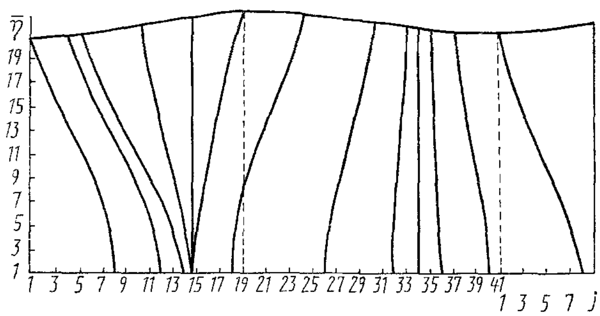

Fig. 4. Distribution of dimensionless pressure along a tube of varying cross section for Re' = 1100; $\bar{l}_0 = 4.13$; $\bar{a} = 0.02$; $\bar{l}_e = 0.45$.

determination of friction. Third, if this is not a calculation error, then in some cases ($\bar{l}_e = 0.75$) a local nozzle ($\bar{l}_e = 0.75$, points 3-7) can appear in the geometric exit cones.

To analyze the velocity profiles in a TVC we show in Fig. 3 the variation of the ratio of maximum velocity at the axis u_m to the mean velocity over the cross section $\bar{u}_m/\bar{u}_{mean} = n_0$. It is well known that in a CT $n_0 = 2$, corresponding to a profile in the form of a quadratic parabola (Poiseuille).

These curves contain quite interesting qualitative information. Thus, the portion with $n_0 > 2$, i.e., where the velocity profiles are more prolonged, can be assumed to be the region in which the velocity gradient is at the wall, and consequently the friction is less than in a CT, while in the portion $n_0 < 2$, on the other hand, the velocity profile is more filled and the friction stress is larger than in CT. We recall that when the quantity n_0 is of order 1.4 and less, the flow becomes turbulent. In Fig. 3 the region with $n_0 > 2$ corresponds approximately in magnitude to $\bar{l}_e$ values, i.e., to exit cone portion lengths.

The difference between largest and smallest n_0 values, equal to Δn_0, corresponds qualitatively to the value of velocity profile deformation. Since the shape hydrodynamic resistance is expressed by energy losses on the deformed velocity profile (they are absent in a CT), then, consequently Δn_0, and, more accurately, the difference in mean n_0 values for n_0 larger and smaller than 2, makes it possible to estimate qualitatively the value of the shape resistance. These differences are not large in Fig. 3, since the amplitude value is low ($\bar{a} = 0.02$). It can be noted, however, that for $\bar{l}_e = 0.85$, Δn_0 is smaller than for $\bar{l}_e = 0.45$, and consequently, the shape resistance at $\bar{l}_e = 0.45$ can be larger than for $\bar{l}_e = 0.85$.

The pressure field was calculated by the corresponding Poisson equation. As an example we show the pressure field (Fig. 4) for $\bar{l}_e = 0.45$; $\bar{a} = 0.02$ and $\bar{l}_0 = 4.13$. It can be verified that at the inlet to the exit cone (cross sections 1-13) the pressure at the wall is lower than at the axis, and consequently $\partial p/\partial r < 0$, at cross section 14 the pressure is constant $\partial p/\partial r = 0$, further on (cross sections 15-33) the derivative changes sign, in cross section 34 it vanishes again, and beyond cross section 34 it becomes negative again. This nature of pressure variation is observed for all other TVC. Two cross sections have always occurred in which $\partial p/\partial r$ vanished. The first of them is in the exit cone portion, and the second – in the nozzle.

We are unaware of studies for nonsymmetric waves. Verification of the calculation algorithm, carried out by comparison with the results obtained for $\bar{l}_e = 0.5$ [5], gave good agreement.

3. Hydrodynamic Resistance. The total hydrodynamic resistance of the TVC was determined by the pressure drop at the corresponding cross sections. In the mathematical model it was obtained from the Poisson equations, and experimentally – by direct measurements.

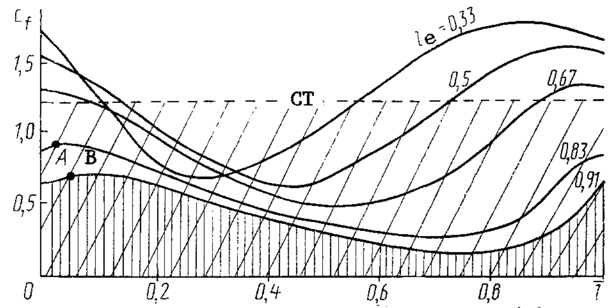

Fig. 5. Distribution of local hydrodynamic resistance friction coefficients c_f over the length of the exit cone–nozzle section for various $\bar{l}_e$ values for Re' = 1100; $\bar{l}_0$ = 4.13; $\bar{a}$ = 0.06.

TABLE 2. Ratio of Total Resistance, Friction and Shape Resistance Coefficients of TVC to CT

Re'	$\bar{\lambda}_f$			$\bar{\lambda}_f$	$\bar{\lambda}_e$
	a=0,01	a=0,02	a=0,03	a=0,03	
500	0,867	0,766	0,665	0,305	0,360
700	0,815	0,668	0,611	0,269	0,343
900	0,740	0,575	0,526	0,252	0,274
1100	0,669	0,503	0,436	0,235	0,201
1300	0,615	0,435	0,367	0,226	0,141
1500	0,561	0,386	0,289	0,210	0,089
1700	0,525	0,351	0,230	0,201	0,029

In the numerical experiment one can determine the isolated friction value and find the shape (pressure) resistance from the velocity fields.

The local friction coefficient c_f is determined by the equation:

$$c_f = \tau_w \Big/ \left(\frac{\rho u^2_{mean}}{2} \right) = \left(\mu \frac{\partial u_{\parallel}}{\partial n} \right) \Big/ \left(\frac{\rho u^2_{mean}}{2} \right),$$

where $u_{\parallel}$ is the local velocity, parallel to the tube wall.

The total friction resistance coefficient λ_f was determined from the equation:

$$\lambda_f = \frac{1}{l_0} \int_0^{l_0} c_f dx.$$

Figure 5 shows the variation in the local friction coefficient c_f at $\bar{a}$ = 0.06; Re' = 1100 and $\bar{l}_0$ = 4.13 for $\bar{l}_e$ varying from 0.33 to 0.91. Also given is the straight line (c_f = 1.21) for the cylindrical tube. It is seen that for small $\bar{l}_e$ (0.33, 0.5, and 0.67) portions occur in a TVC at the nozzle and beginning of the exit cone regions, in which c_f, and consequently the friction stress as well, are larger than in a cylindrical tube. It is easily noticed that the length of these portions decreases with *increasing* $\bar{l}_e$. At $\bar{l}_e$ = 0.33 it is nearly $0.5\bar{l}_0$, at $\bar{l}_e$ = 0.5 it increases to approximately $0.6\bar{l}_0$, and at $\bar{l}_e$ = 0.67 – to $0.75\bar{l}_0$. Further increase in $\bar{l}_e$ (0.83 and 0.91) leads to the whole TVC region having a c_f smaller than for the cylindrical tube.

The ratio of areas, shaded by vertical lines, to the areas shaded by the inclined lines (Fig. 5) shows what part distinguishes the friction force of the TVC at $\bar{l}_e$ = 0.91 from the friction force for a cylindrical tube. It equals 0.67. At $\bar{l}_e$ = 0.75 it equals 0.83, while for $\bar{l}_e$ = 0.67, 0.5, and 0.33 it is 1.26, 1.47, and 1.72, respectively.

The effect of the Re' number on the total resistance coefficient of a TVC (friction and shape), referenced to the corresponding coefficient of a CT, is given in Table 2 within the Re' limits from 500 to 1700. It is seen that for $\bar{l}_e$ = 0.75 this quantity decreases substantially with increasing Re' for all three amplitudes: for $\bar{a}$ = 0.01 – from 0.867 to 0.525, and for $\bar{a}$ = 0.03 – from 0.665 to 0.23.

In the same table we show separately for $\bar{a}$ = 0.03 the variation in the corresponding ratio of shape and friction coefficients. It is seen that for the mentioned increase in Re' the first decreases more quickly (from 0.36 to 0.03) than the second (0.3 and 0.2).

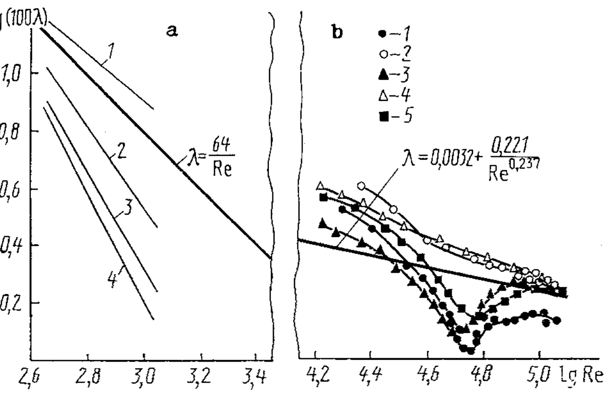

Fig. 6. The dependence of log(100λ) on logRe: a) laminar flow, calculated for $\bar{a}$ = 0.03; $\bar{l}_0$ = 4.13; 1) $\bar{l}_e$ = 0.85; 2) $\bar{l}_e$ = 0.8; 3) 0.625; 4) 0.75; b) turbulent flow, experiment 1) $\bar{a}$ = 0.05, $\bar{l}_0$ = 2.06, $\bar{l}_e$ = 0.75; 2) 0.05, 2.06 and 0.25; 3) 0.05, 4.13, and 0.75; 4) 0.05, 4.13 and 0.25; 5) 0.09, 4.13, and 0.5.

To verify the results of the mathematical simulation we carried out a physical experiment on a TVC with $\bar{l}_e$ = 0.25, 0.5, and 0.75; $\bar{a}$ = 0.05 and 0.09; $\bar{l}_0$ = 2.06; 4.13 for Re = $(0.15\text{-}1.08)\cdot 10^5$. Figure 6 shows the commonly accepted dependences of log(100λ) on logRe for circular cylindrical tubes for laminar and turbulent motions and the curves obtained from theoretical calculations for small Re and from experiment at large Re for a TVC.

From analyzing the dependences shown in Fig. 6 it follows that by varying the geometric characteristics of a TVC one can obtain both an increase and a decrease in the hydrodynamic resistance coefficients in comparison with an ordinary circular cylinder. In this case these variations can be carried out over the whole range of Re numbers, i.e., for both laminar and turbulent motions.

The maximum reduction in the mathematical simulation reached a value of 5 times (Table 2), while the experimental value was 2 times less than the resistance of a cylindrical tube, corresponding to loss reductions by more than 60 and 40%. We recall that small additions of polymers and SAM can affect the resistance only for turbulent motion.

The decrease in resistance is a major problem for tubes performing transport functions. For heat exchangers and tubes designed for chemical technology optimal transport coefficients can usually be obtained by increasing the resistance coefficients. Consequently, consistently with the Reynolds analogy, wavy tubes make it possible to control transport processes in general, i.e., not only the hydrodynamic resistance, but also heat and mass transfer.

Contemporary technology does not allow one to create nearly natural surfaces, such as blood vessels (compliant, easily varying their shape under the action of flow, and guaranteeing physicochemical and physiological processes), which create conditions for long duration (dozens of years) of livelihood activity of organisms.

The first step in approaching more perfect natural processes – the creation of tubes with rigid walls, but with varying cross section with length – is a real problem, and the energy saving effect requires extensive application of asymmetric wavy tubes.

NOTATION

Here the geometric parameters are: a is the amplitude in m, l_e is the length of the exit cone portion in m, l_0 is the wave length in m, r_0 is the radius of a straight tube, equal to the mean radius of a tube with varying cross section, in m, x and r are the longitudinal and radial coordinates in m, $r_t(x)$ is the radius of a tube of varying cross section in m, $\bar{a} = a/r_0$ is the dimensionless amplitude, $\bar{l}_e = l_e/l_0$ is the dimensionless length of the exit cone, $\bar{l}_0 = l_0/2r_0$ is the dimensionless wave length, $\bar{x} = 2\pi x/l$ is the dimensionless longitudinal coordinate, $\bar{r} = r/r_0$ is the dimensionless radial coordinate, $\bar{r}_t = r_t/r_0$ is the dimensionless radius of a tube of varying cross section, and $\bar{\eta} = r/r_t$ is the dimensionless radial coordinate, introduced in transition to the rectangular region. The hydrodynamic parameters are: u, v are the longitudinal and transverse components of the velocity vector in m/sec, ρ is the density in kg/m^3, ν is the kinematic viscosity coefficient in m^2/sec, μ is the dynamic viscosity coefficient in kg/(m·sec), ω is the vorticity in 1/sec, ψ is the stream function in m^3/sec, Q is the bulk discharge in m^3/sec, u_{mean} is the mean discharge velocity in m/sec, p is the pressure in N/m^2, $\bar{\psi}$ is the dimensionless stream function, $\bar{\omega}$

is the dimensionless vorticity, $\bar{u} = ur_0^2/Q$ is the dimensionless longitudinal component of the velocity vector, $\bar{v} = vr_0^2/Q$ is the dimensionless radial component of the velocity vector, $\bar{p} = p/(\rho u_{mean}^2)$ is the dimensionless pressure, $\Delta\bar{p}_0$ is the dimensionless pressure drop in a CT, $\Delta\bar{p}$ is the dimensionless pressure drop in a TVC, $C_f = \tau_w/(\rho u_{mean}^2/2)$ is the local friction resistance coefficient, λ_f is the friction resistance coefficient, λ_e is the pressure resistance coefficient, λ_t is the total resistance coefficient in a TVC, λ_0 is the resistance of a CT, $\bar{\lambda}_t = \lambda_t/\lambda_0$ is the ratio of the total TVC resistance to the CT resistance, $\bar{\lambda}_f = \lambda_f/\lambda_0$ is the ratio of the TVC friction resistance to the CT resistance, and $\bar{\lambda}_e = \lambda_e/\lambda_0$ denotes the ratio of the TVC pressure resistance to the CT resistance.

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DISTRIBUTION OF VOLUME-AVERAGED PARAMETERS OF VORTEX FLOW OVER THE ENERGY SEPARATION CHAMBER OF A VORTEX TUBE WITH SUPPLEMENTED FLOW

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The profiles of averaged circular and axial velocity components, temperature, and pressure are measured in the axisymmetric channel of the energy separation chamber of a vortex tube with the introduction of additional flow in the near-axis zone on the side cross section where throttle is located.

In a series of engineering devices (vortex burners, plasma production apparatus, vortex igniters, and projected power equipment), the preferred, and sometimes the only method for controlling the mass and heat transfer is to sharply whirl the flow as it flows along an axisymmetric channel. Vortex flow in the energy separation chambers of vortex tubes is one of the most complex and incompletely understood methods. While the gas dynamics of counterflow separation vortex tubes is known [1, 2], there are no publications of such investigations of double-contour tubes. At the same time, knowledge of the nature of the distribution of thermodynamics and kinematic flow parameters is important for developing an adequate physical and mathematical model, in order to perfect further the energy separation processes in these tubes. This is even more important because today these tubes have the highest adiabatic efficiency, as computed from the overall characteristics of the process for cooling part of the gas. In some tubes the cooled gas mass flow exceeds the initial flow [2, 3].

The investigation was conducted on a unit whose working section was a conical axisymmetric vortex tube with a minimum diameter of the separation chamber $d_1 = 0.03$ m and with a cone angle $\gamma = 15°$; the relative length of the energy separation chamber was $\bar{l} = L/d_1 = 9$. The relative area of the nozzle input $\bar{F}_n = F_n/F_{tr}$ was measured from 0.02 to 0.1 in

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